of the corresponding uniform panel. The particular values shown in Figs. 2 and 3, however, may not be extremely accurate due to the nature of the approximate method and the two-mode deflection assumption. It is hoped, however, that these results may provide a starting point for further analysis and design by other methods.

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Dark-Pause Measurements in a High-Pressure Arc Discharge

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THE experiments on exploding wires are fairly complete,[1] but the data on dwell and restrike characteristics of a confined arc in a closed chamber are fragmentary, particularly at pressures above 1 atm. In this paper, we examine the dwell or dark-pause data compiled through 2 years of tests of an arc-heated, shock tube driver at the Ames Research Center.[2,3] In this driver, helium (or other) gas under several atmospheres of pressure in the arc chamber (9.5 cm diam) is heated at constant volume by an electrical arc discharge. Energy for the arc is supplied from a one megajoule, 40 kv capacitor bank. On the basis of our data, it is possible to give at least a qualitative interpretation of the dwell period. The data cover a wide range of pressures and gases for three different arc lengths—76, 137, and 290 cm.

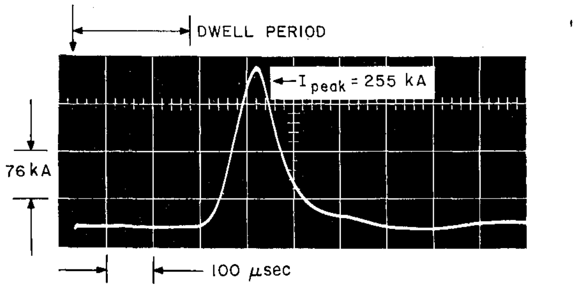

Fig. 1 Typical current trace for 290-cm arc length, driver load pressure of 10.2 atm helium, tungsten trigger wire of 0.127-mm diam, preset voltage of 40 kv.

To initiate an arc discharge between widely spaced electrodes at high gas pressures requires the use of an ignition wire bridging the electrode gap. Generally, the wire is stretched or drawn between the electrodes. The general picture of how the trigger wire leads to the eventual arc discharge is described as follows: the initial current surge instantly explodes the wire from its metallic state by ohmic heating to a nonconducting column of hot aerosols at high pressure. The abrupt decrease in the electrical conductivity stops the capacitor discharge with no apparent change in the bank voltage. As a result of the increase in pressure, the column expands and acts on the surrounding gas like a piston, sending into it a cylindrical shock wave. With the gasdynamic expansion, the pressure (and density) of the vapor decreases in the central region of symmetry. When the pressure reaches a critical value,[4] avalanche breakdown starts, and the arc strike occurs. The time required for the central core region to reach this critical value of pressure is defined as the dwell period. If the critical value is not reached, the arc will not occur and the system will remain in an open-circuit condition.

The relationship between the breakdown voltage and the product of (pressure) x (electrode spacing) is generally known as Paschen's law. This relationship is discussed in textbooks,[5] together with information that can be used to calculate the minimum breakdown voltage for a particular electrode arrangement, for a given gas and pressure. Under the range of experimental conditions employed, Paschen's law is used to interpret the data together with concepts from cylindrical blast wave theory.

A typical current waveform for an arc length of 290 cm is shown in Fig. 1. From this oscillogram, and others like it, the dwell period was measured as the interval from the wire explosion to the start of the rise of the arc current. The dwell period is terminated once the restrike current flow starts and, thus, the dwell characteristics can be evaluated without regard to subsequent shock tube events; i.e., diaphragm opening, shock-wave development, etc. While the trigger wire was exploded in times apparently less than 1 microsecond, the vaporized metal column does not instantaneously induce the arc strike. With a 76-cm arc length, the duration of the period from explosion to the start of the arc discharge varied from 900 μsec for an 8 kv preset voltage to about 20 μsec at 40-kv preset. Long dwell periods were noted also for the 137- and 290-cm arc lengths. After collecting the dwell data compiled through something like 500 firings, it was found that, if the data were presented under one driver pressure condition, with the same trigger wire arrangement, the dwell period is a function only of the initial field intensity (i.e., preset voltage/arc length). As summarized in Fig. 2, the results for three different arc lengths fall on a single line on the log-log plot. The load pressures of the gases in the three driver chambers were kept at 18.5 atm and the same type of straight trigger wire was used in each—a 0.127-mm-diam tungsten wire. The gases and gas compositions tested were helium, argon, nitrogen, a mixture of 1-atm air plus helium, and mixture ratios of 10–50% argon in helium. Dwell periods for helium (and He/Ar mixtures) were measured from repeated test in the facility, and the run-to-run variation was small (±4%).

Voltage and current records were obtained also over a range of driver pressure operation for two of the driver lengths with the same trigger wire arrangement as noted in connection with Fig. 2. As summarized in Fig. 3, these dwell data show an increasing duration with increasing gas pressure. A

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further noticeable feature in Fig. 3 is that the arc did not restrike above a chamber pressure of about 28 atm for the 290-cm length. As an aside, with the 137-cm length (at 40 kv) the maximum pressure for restrike was increased to about 34 atm.

The exploding-wire, dwell, and arc-discharge processes are very complicated phenomena.[6,7] The experimental evidence for the parameter range covered in this paper is considered verification of the concept that the dwell period is a result of the pumping time required to attain a certain low pressure in the center region of the exploded wire. Blast-wave theory[8] shows the development of the rarefied region. However, the wave interaction in this central core is complicated, particularly if consideration is given to the phase change of the wire material. Although attempts to calculate a particular dwell duration have been unsuccessful, the trend of the experimental results is considered to be in general conformation with the cylindrical blast case. For example, the dwell period increases with load pressure (Fig. 3), indicating a slower pumping speed (slower shock wave) as would be expected for a constant energy input driving a denser gas. A second example is the relative consistency of the core pressure, estimated in the following manner. It was found that there is a minimum voltage required to induce an arc strike.[3] If we assume that this voltage is the breakdown voltage, then it is a function of (pressure) x (spacing) alone—Paschen's law. With the 76, 137, and 290-cm arc lengths, these voltages were about 8, 16.5 and 34 kv, respectively. A Paschen type of analysis indicated that the (center region) pressures for the three arc lengths were similar and quite low, only about 45 torr. This is approximately 10^{-3} of the initial gas load pressure (18.5 atm). With increasing voltage, the Paschen relationship indicates an increasing pressure for breakdown. For the 76-cm arc at 40 kv (v/cm of 521), the breakdown pressure is 302 torr. The higher the critical value of pressure, the shorter the period of time for the wave system (directed towards the center region) to reach this prerequisite pressure. Thus, it appears that the decrease in dwell period with field intensity (Fig. 2) could be attributed to breakdown voltage characteristics based on Paschen's Law.

Experimentation has shown that only certain types of wire materials produced the arc discharge, although all wire materials tested were exploded within the driver chamber. Tungsten, titanium, chromel-C, nickel, and aluminum wires initiated an arc strike. Copper and manganin did not.

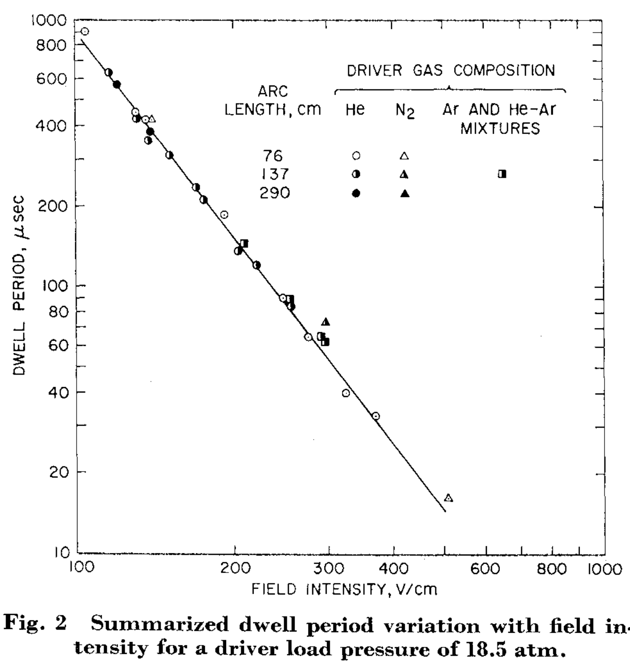

Fig. 2 Summarized dwell period variation with field intensity for a driver load pressure of 18.5 atm.

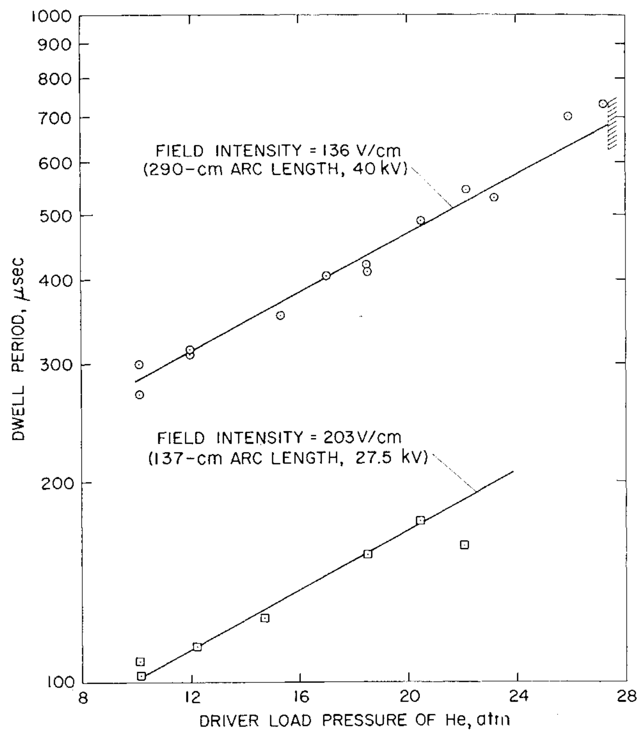

Fig. 3 Dwell period variation with driver load pressure.

Parenthetically, it is this nonrestrike property of copper materials that make them well suited for application as expulsion-type fuses. With tungsten, titanium, and chromel-C wires, the dwell periods were, to all practical purposes, the same. Wire diameter (0.10 to 0.25 mm) made no discernible difference in the period. However, at the test condition of a field intensity of 205 v/cm, the use of nickel and aluminum wires resulted in a shorter dwell period of about 110- and 50-μsec, respectively. This is compared to the 155-μsec time noted from Fig. 2. Although the different types of wire materials were not tested in the different gas environments, it is speculated that generally similar characteristics curves would be obtained for the various wires.

One other comment on the trigger wire material is in order. The power generated to create the cylindrical shock depends on how much or how well energy can be transferred into the wire before it explodes. Materials with a high boiling point and high heat of vaporization are capable of storing more energy than those of low boiling point and low heat of vaporization. Transition elements (W, Ni, Fe, and Ti) fall into the former category, while Cu and Al are classified in the later. Thus, a mechanism[9] is indicated that accounts in part for the different materials and (except for aluminum) is in agreement with experimental trends noted herein. If the shock strength is too low, the wave motion probably is damped by viscous action before the central region can be "pumped" to the necessary low pressure for restrike. The concept of Paschen's law, combined with knowledge of the differences of materials in the wire explosion, appears to offer a physical accounting of the dwell period. It is hoped that a sound theoretical calculation combining Paschen's relationship, wire material properties, and cylindrical shock behavior will be formulated in the near future.

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An Exact Differential Method to Determine Liapunov Stability

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In this Note a simple method is presented to find the time derivative of a Liapunov function[1,2,3] in order to test the asymptotic stability of an autonomous system. The concepts[1,2,3] of Liapunov's function are not presented here. However, it may be mentioned that if the Liapunov function in the study of motion of a system is positive definite and its total rate of change with respect to time is negative definite or negative semidefinite, then the motion of the system is considered to be asymptotically stable.

The Liapunov function $V(x)$ is a scalar, and it is not unique. The line integral[4] between two points is independent of the path of integration, and it is assumed that the curl grad V is equal to zero.

In the analysis of the motion of a system, we obtain a system of equations of motion. Utilizing the state variable analysis,[2] we may reduce the equations of motion into a set of state variable equations. Without solving these equations, we may determine the asymptotic stability of the motion by employing the Liapunov stability criteria in the following formulations:

$$\text{Let } V = \int_{p_1}^{p_2} d[\Sigma x_i^2 x_j^2]; \quad i,j = 1,2, \ldots n \tag{1}$$

For brevity, we consider 3-dimensional case in Eq. (1) and obtain

$$V = \int_{p_1}^{p_2} [(4x_1^3 + 4x_1x_2^2 + 4x_1x_3^2)dx_1 + (4x_2^3 + 4x_2x_3^2 + 4x_2x_1^2)dx_2 + (4x_3^3 + 4x_3x_1^2 + 4x_3x_2^2)dx_3] \tag{2}$$

Recalling that the integral of a gradient is independent of the path of integration, we write

$$V = \int_{p_1}^{p_2} \left[\frac{\partial V}{\partial x_1} dx_1 + \frac{\partial V}{\partial x_2} dx_2 + \ldots + \frac{\partial V}{\partial x_n} dx_n\right] \tag{3}$$

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For 3-dimensional case

$$V = \int_{p_1}^{p_2} \left[\frac{\partial V}{\partial x_1} dx_1 + \frac{\partial V}{\partial x_2} dx_2 + \frac{\partial V}{\partial x_3} dx_3\right] \tag{4}$$

Using the exactness criteria, we get from Eqs. (2) and (4),

$$\begin{aligned} \partial V/\partial x_1 &= 4x_1^3 + 4x_1x_2^2 + 4x_1x_3^2 \\ \partial V/\partial x_2 &= 4x_2^3 + 4x_2x_3^2 + 4x_2x_1^2 \\ \partial V/\partial x_3 &= 4x_3^3 + 4x_3x_1^2 + 4x_3x_2^2 \end{aligned} \tag{5}$$

And the time derivative $\dot{V}(x)$ is

$$dV/dt = \Sigma(\partial V/\partial x_i)\dot{x}_i; \quad i = 1,2, \ldots n \tag{6}$$

The procedures are explained by an example.

Let us consider the system

$$\dot{x}_1 = -x_2 - x_1^3; \quad \dot{x}_2 = x_1 - x_2 \tag{7}$$

using Eqs. (5) and (6), we get for 2-dimensional case

$$\begin{aligned} dV/dt &= (4x_1^3 + 4x_1x_2^2)(-x_2 - x_1^3) + (4x_2^3 + 4x_2x_1^2)(x_1 - x_2) \\ &= -(4x_1^6 + 4x_2^4 + 4x_1^4x_2^2 + 4x_2^2x_1^2) \end{aligned}$$

which is negative definite. Therefore, the system [Eq. (7)] is asymptotically stable.

As far as line integral and the assumptions are concerned, this technique has similarity only with the existing variable-gradient method.[2] An iterative procedure is required to determine the coefficients of the gradient matrix in the variable-gradient method. Although the partial derivative of an arbitrary positive definite function $V(x)$ with respect to the state variables may lead to some results, the exactness approach in the field of curl grad $V = 0$ may establish a mathematical criteria for a methodology.

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Vibration Analysis by Differential Holographic Interferometry

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A NEW double-pulse method of holographic interferometry is proposed. This technique intends holography to permit measurements on large, noisy subjects vibrating to

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